Complete the square. $\frac{b}{2a}$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

6-5 The Quadratic Formula and the Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex 1

$3x^2 + x - 1 = 0$

$a = 3$
$b = 1$
$c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-1 \pm \sqrt{13}}{6}$

ex 2

$5x^2 + 8 = -12x$

$5x^2 + 12x + 8 = 0$

$a = 5$
$b = 12$
$c = 8$

$x = \frac{-6 \pm 2i}{5}$

The Discriminant

$$D = b^2 - 4ac$$

Determines the nature of the roots.

Three Cases

I. $D > 0$

2 Real (unequal) roots

II. $D = 0$

double $\mathbb{R}$ root

III. $D < 0$

2 imaginary roots

Rational

If D is a perfect square & a, b, + c are rational

Ex

$x^2 - 8x + 5 = 0$

$D = b^2 - 4ac$

$64 - 4(1)(5)$

$D = 44$

2 $\mathbb{R}$ irrational roots

Also able to determine if the roots are rational or irrational.

Rational
a, b, & c must be rational and D must be a perfect square (Real)

ex
$x^2 + 10x + 25 = 0$

$D = 100 - 4(1)(25)$
$D = 0$
double real rational root

ex
$x^2 - 4x + 13 = 0$

$D = 16 - 4(1)(13)$
$16 - 52$
$= -36$
2 imaginary roots

Determine as much as you can about the roots:

1. $y^2 - 3y - 1 = 0$ $D = 13$ 2 $\mathbb{R}$ irr.
2. $3a^2 - 10a = -11$ $D = -32$ 2 imaginary
3. $5x^2 + 2\sqrt{10}x + 2 = 0$ $D = 0$ double $\mathbb{R}$ irrational
4. $3b^2 = 14b + 24$

$D = 484$ 2 $\mathbb{R}$ rat'l

Find the value for k such that there are 2 imaginary roots
ex
$5x^2 - 2x + k = 0$ $D < 0$

$4 - 4(5)(k) < 0$
$4 - 20k < 0$
$k > \frac{1}{5}$

Find the value for k such that there is a double root
ex
$3x^2 + 2x + k = 0$ $D = 0$

$4 - 4(3)(k) = 0$
$4 - 12k = 0$
$4 = 12k$
$k = \frac{1}{3}$

p318
15-25 odd and 45a
(not #21)